Modelling Price Dynamics as the Outcome of Interaction between Heterogeneous Agents

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Abstract: If the market price is considered to be the outcome of the interaction of heterogeneous agents, there may be an important effect of lagged absolute returns on price dynamics. In order to illustrate this effect a simple model of heterogeneous traders is introduced which, although minimalist by construction, does produce realist price dynamics. Simple changes in dynamics conditional on lagged returns in various daily equity indices are assessed by means of smooth transition autoregressive models. The results are reasonably supportive of the conclusion that such interaction effects may be present in actual market data.

Keywords: Heterogeneous agents; Price dynamics; Smooth transition autoregressive models

1. INTRODUCTION

Many financial markets exhibit a range of similar empirical characteristics. It is well established that returns from financial assets have non-normal distributions, exhibit volatility clustering and contain some nonlinear structure. A recent approach to account for these facts has been to describe the market as the observed outcome of many interacting agents with heterogeneous behavioural patterns. The notion of heterogeneity stems from Black [1986] who highlights the importance of noise in financial markets. While noise may be considered as low quality information regarding value, its role in motivating trading to ensure liquid markets is imperative. Whereas noise in the usual statistical sense is considered to be the outcome of a particular random sequence, noise in the informational sense is not necessarily random in nature. Much noise trading activity that occurs is not random and is the result of mechanistic trading rules or fundamental psychological biases in the decision making process. Such behaviour has become known as feedback trading. This notion of feedback behaviour, in various forms, has been included in many (some quite complex) models of agent interaction to describe the microstructure perspective of trading in a financial market. Such models of interaction include Chiarella [1992], Day and Huang [1990], De Grauwe et al. [1993] and Lux [1998]. The motivation for these models has been to develop plausible accounts for the observed dynamics of financial markets. To this end, they have been quite successful.

One problem with these recent contributions is the increasing level of complexity used to describe feedback behaviour. The model proposed in this paper, however, seeks to be completely transparent and make minimal assumptions about the information sets required by traders. To this end, the starting point is taken to be Cutler et al. [1990] who define the behaviour (excess demand for assets) of feedback traders as a linear function of lagged returns. While the information set utilised is straightforward, a simple linear reaction to returns history may not truly reflect the mechanisms underlying feedback behaviour. Technical analysis, as one example of feedback trading, suggests the use of trading ranges, support and resistance points and breakout patterns, to name but a few, all of which may produce non-linear reactions to price movements. The concept of a threshold in representing the trading decision of a feedback trader is useful when considering the mechanistic rules underlying strategies that generate feedback patterns. This paper differs in its description of feedback behaviour, in the sense that it allows for a probabilistic assessment of the trading decision on the part of feedback traders, incorporating the notion of thresholds and a link by which past returns influence decision making.

If decision-making is influenced by market conditions then it may be expected that measures of current returns, as a reflection of the amount of information arriving into the market, may have an effect on future price dynamics. Ensuing interaction between agents produces price changes exhibiting periods of both calm and. This behaviour is not subordinated to any underlying information arrival process as suggested by Clark [1973] and examined by Lamoureux and Lastrapes [1990] with trading volume was used as the proxy for information flow. Outcomes are solely due to the decision making process employed by heterogeneous groups of traders. It is their behaviour, by reacting to price movements that propagates volatility and creates persistence in the observable when there is none in the underlying.

Section 2 of the paper will outline the behavioural rules underlying the model of interaction and discuss the ensuing dynamics. Section 3 discusses the impact of past returns on current mean dynamics from both the model of interaction and a number of equity indices. Section 4 contains concluding comments.

2. MODELLING AGENT INTERACTION

2.1 Feedback traders

This model specifies feedback behaviour based solely on past price movements, but in such a way that the herding element is generated. In this model 100 individual feedback traders are assumed to exist and each is allocated a unique price, $\tilde{p}_{i,t}$ $i = 1 \ldots 100$, at which their last trade occurred. This price is initially generated randomly as a normal deviate with mean given by the starting price, p_0, and is updated by each individual anytime they decide to trade.

The trading decision for feedback trader i is based on the absolute difference between the current price and the historic trade price, $|p_t - \tilde{p}_{i,t}|$, in a probabilistic manner as follows

$$D^i_{f,t} = \begin{cases} \pm\kappa & \text{if } m_t \cdot |p_t - \tilde{p}_{i,t}| \geq U[0,1] \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where κ is a constant demand, whose sign matches that of $p_t - \tilde{p}_{i,t}$ and m_t is a parameter that determines the width of the threshold within which trading is uncertain. For example, if $m_t = 20$ feedback trader i will trade with certainty if $|p_t - \tilde{p}_{i,t}| \geq 0.05$. In the interim, a trade is more likely the greater this absolute difference becomes. The beauty of this simple model is that it allows both longer-term price swings and very current returns to impact upon demand. The former effect is captured by the central role of $p_t - \tilde{p}_{i,t}$ in the decision rule, and the latter effect is obtained by the changing threshold parameter.

It remains to say a few words about how the threshold parameter is updated. The width of a trader's threshold reflects the importance placed on $p_t - \tilde{p}_{i,t}$ and the belief that there is persistence in price movements. At any given point in time each feedback trader assesses the relative magnitude of the current return, r_t , by means of a standardized style measure,

$$r_{s,t} = r_t / \sigma_r \quad (2)$$

where σ_r represents a prior of the volatility of returns and is set in accordance with other parameters from within the model. If $r_t < \alpha$, a given constant and the sign of r_t matches that of $p_t - \tilde{p}_{i,t}$ a narrower threshold is used in trading decisions for the upcoming period, that is to say $m_{t+1} > m_t$. This increase results in a narrower bound past which trading is certain, a feature which reflects an increased state of anxiety in feedback traders and serves to increase current volatility. While the criteria used to assess and weight information are arbitrary, they serve to highlight the possibility that changes in the flow of information may change the behaviour of individuals in the market and thus overall dynamics.

2.2 Fundamental traders

In common with the previous literature, traders acting on information regarding the fundamental value of assets are also considered. Such participants represent the process of arbitrage in the marketplace. Based on current price and their perception of fundamental value, they trade with the view that a divergence between market and fundamental value will be eliminated. Their strategy is clearly one of buying (selling) when the current price, p_t, is below (above) perceived fundamental value $p_{f,t}$ in the belief that this mispricing will be eliminated. As a reflection of the uncertainty surrounding such a strategy due to noise trader risk, as discussed in De Long et al. [1990], demand from fundamentalists will not appear until prices have drifted sufficiently from underlying value where expected returns from the reversion is sufficient to overcome noise trader risk. Excess demand for an asset from such a strategy will take the following form

$$D_{r,t} = \begin{cases} -\beta(p_t - p_{f,t}) & \text{if } \dfrac{|p_t - p_{f,t}|}{p_{f,t}} \geq \lambda \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

Here λ represents the threshold past which fundamentalists deem the strategy to be worthwhile in the sense that its expected return is sufficient compensation for the risk due to the action of feedback traders.

2.3 Price dynamics

Assuming a constant supply of the stock so that price adjusts to eliminate excess demand, market price evolves according to

$$\Delta p_t = D_{r,t} + \sum_{i=1}^{100} D^i_{f,t} + D_{n,t} \qquad (4)$$

where $D_n \sim N(0,\sigma_n^2)$ is included to represent random trading not linked to information and possibly due to liquidity or other various idiosyncratic requirements.

Figure 1 depicts a typical price path of 2000 observations, generated with $p_{f,t}$ evolving as a random walk. Parameters chosen for this simulation are $\kappa = 0.0025$, $\sigma_n = 0.0015$, $\sigma_r = 0.001$, $\alpha = 0.5$, $\beta = 0.05$ and $\lambda = 0.15$. The effects of altering various parameters will discussed in a later section subsequent to an examination of the resultant dynamics.

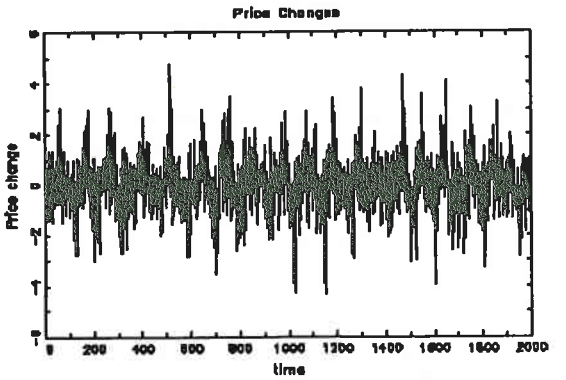

Figure 1. Evolution of simulated returns.

It is clear from Figure 1 that the interaction of the groups of traders generates a system where prices are driven away from value and then revert to the fundamental. Periods of relative calm are followed by breakouts in prices as particular signals from either group dominate.

Two common empirical facts of financial time series are that the distribution of returns exhibit excess kurtosis and high persistence in volatility. Figure 2 indicates that price changes from this system do in fact show the former property with the distribution having a sharper peak around the mean and somewhat heavier tails than a normal distribution. The persistence in volatility may be verified by estimating a simple model of conditional variance. The important point to note here, however, is that the persistence in variance is now explicity related to the dynamic pricing mechanism. It is the changes in mean dynamics due to the interaction of heterogeneous traders which accounts for the conditional volatility. An appropriate model of the dynamics should account for the heteroskedasticity.

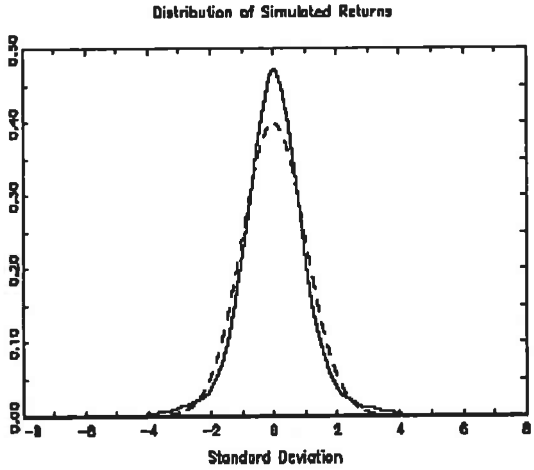

Figure 2. Non-parametric density estimate of the standardized simulated returns (solid curve) and standard normal distribution (dashed curve).

While these results are generated by the interaction of the particular behavioural rules outlined previously and for the assumed parameter values, they do indicate reasonably realistic price dynamics. The impact of varying some of these parameters is now outlined. Increasing α has little impact on dynamics, the distribution of returns becomes somewhat less peaked and prices become more regular as traders are trading more frequently and in a more correlated manner. Conversely, when α decreases, returns approach a normal distribution where prices become more random as less coupling of traders occurs, due to wider thresholds in the decision-making process. Increases in λ have no discernible effect, while decreases lead to a slightly less peaked returns distribution and more regular price movements. The parameters κ and β are set to ensure a stable system. If both are increased this simply leads to deviations from value being created and extinguished at a higher frequency as the strength of the signals increase. In general, price dynamics appear to be reasonably robust to changes in the parameter values.

Outcomes from the interaction of heterogeneous trading rules do reflect characteristics observed in financial asset returns. Price changes from this simulated model do exhibit non-normal distributions with excess kurtosis and high levels of persistence in conditional volatility. The returns appear to exhibit periods of both calm and calamity without subordination explanations. Such observations themselves are not of great interest in that previous models also produce such behaviour. The novel point here is that such dynamics may be produced when feedback effects are influenced by the magnitude of past returns and thus a link to mean dynamics would be expected. The following section investigates this possibility.

3. MEAN DYNAMICS

In general, models of volatility describe conditional volatility of returns, σ_t^2, to be some function of the magnitude of past returns. The exact form of this dependence has been the subject of a vast literature. Based on the realistic behaviour observed by examining the simulated returns earlier, it is suggested that the magnitude of past returns should have an impact not only on volatility but also on mean dynamics as it impinges upon the decision making of individuals in the market. To capture this effect, an approach that allows the link between past returns and observed dependency is required. For this purpose, the smooth transition framework of Teräsvirta [1994] is utilised where past returns influence current levels of dependence. The focus here will be on the Logistic Smooth Transition Autoregressive (LSTAR) model. The LSTAR approach is chosen in preference to the related Exponential STAR in this situation as changes in the magnitude of returns are deemed to have an abrupt effect on dynamics. In the limit the LSTAR model is capable of approximating a Heaviside Function at the point at which the change occurs and this characteristic is deemed appealing in this particular application where the detection such abrupt changes is paramount. The model takes the following form

$$y_t = \pi_{10} + \pi_1 ' w_t + (\pi_{20} + \pi_2 ' w_t) \cdot F(y_{t-d}) + u_t \quad (5)$$

where

$$F(y_{t-d}) = \left[1 + \exp\left(-\phi\left(y_{t-d} - c\right)\right)^{-1}\right] \quad (6)$$

The parameter d is the delay parameter and its choice determines the lag which is selected as the state variable. The error term, $u_t \sim NID(0, \sigma_u^2)$, is assumed homoskedastic once the mean dynamics account for the dependence on past returns in the nonlinear way suggested by the model and captured by the two-state smooth transition model. Note that $\pi_j = (\pi_{j1} \ldots \pi_{jp})'$ for $j=1,2$ and the state vector w_t is given by $(y_{t-1} \ldots y_{t-d})'$. The value of the state variable influences the transition function $F(\cdot)$ and therefore the contribution of π_{20} and π_2'to the dependence in the $\{y\}$ series. Being based on a logistic function, the transition function essentially switches the impact of these parameters on or off. As the model of interaction suggests the magnitude of returns should influence observed dependency the state variable used here is $|y_{t-d}|$ although, for ease of reference, the absolute value notation will not be explicit. Note that the choice of using the absolute value of returns as the state variable is driven by the theoretical model and to enable the magnitude of returns to impact on price dynamics. The effect of this choice on asymptotic properties of estimates obtained from the LSTAR model will be the subject of future research.

The estimation of the model begins with the test for general non-linearity suggested by Teräsvirta [1994] that is based on the following model,

$$y_t = \gamma_0 + \gamma_1 ' w_t + \gamma_2 '(w_t y_{t-d}) + \gamma_3 '(w_t y_{t-d})^2 + \gamma_4 '(w_t y_{t-d})^3 + \eta_t$$

and is a test of the null hypothesis of $\gamma_2 = \gamma_3 = \gamma_4 = 0$. Based on a determined value for p, d is chosen by taking that value of the parameter which yields the strongest rejection of the null hypothesis. It is then possible to estimate the model by minimising the residual sum of squares function with respect to the parameter vector.

Simulated returns are examined with an AR(4) specification appearing to be a sufficient lag structure. Based on this value for p, the general test for non-linearity is conducted and $d=1$ is found to be the optimal delay parameter. Equation (5) was estimated, with the parameter ϕ being scaled by $\hat{\sigma}^2_{yt}$.

Table 1 outlines the results for this sample for slope parameters only, since both intercept terms were found to be statistically insignificant.

Table 1. Estimates of the slope parameters of an LSTAR model for simulated returns with associated QML standard errors.

	Parameter Estimates for LSTAR model		
	Estimate	**Parameter**	**Estimate**
π_{11}	0.2935 (0.037)	π_{21}	-0.1418 (20.11)
π_{12}	0.0573 (0.079)	π_{22}	-0.3941 (0.243)
π_{13}	0.0083 (0.193)	π_{23}	-0.4974 (0.216)
π_{14}	0.0073 (0.070)	π_{24}	-0.4442 (0.059)
ϕ	289.95 (2076)	c	0.2132 (0.009)

The positive estimate of ϕ means that $F(y_{t-d})\approx 1$ for $|y_{t-1}|<0.2132$ $F(y_{t-d})\approx 0$ for $|y_{t-1}|>0.2132$. In effect this means that there is an abrupt change in dynamics due to changes in $|y_{t-1}|$, in the vicinity of the critical value 0.2132. When lagged returns are relatively small in magnitude, dependence on past returns appears to be negative. This effect disappears when the converse is true.

It is now possible to compare a number of equity index behaviour to that of the simulated returns. Indices considered are as follows: the S\&P 500, 12740 observations from 16 January 1950 to 13 June 2000; the FTSE 100, 3899 observations from 6 January 1984 to 28 February 2000; the DJIA, 12612 observations, 3 January 1951 to 17 October 2000; and the Australian SPI 5360 observations, 2 January 1980 to 6 March 2001. All are parsimoniously defined by an AR(2) process and have an optimal value of $d=1$ with the exception of the SPI index for which an AR(1) is sufficient. Results for estimating equation (5) for all the indices are reported in Table 2.

It appears that all of the indices examined here do in fact exhibit the same behaviour to varying degrees. That is, when lagged returns are relatively small in magnitude, dependence appears to be dominated by a negative relationship while this effect disappears when the converse is true. Qualitatively the results appear different for the S&P and DJIA on one hand, and the FTSE and the SPI indices on the other. For the FTSE in particular, the statistical significance of the results is of concern with the two crucial parameters, π_{21} and ϕ, both being insignificant. For the SPI it is only the estimate of ϕ which is poorly resolved, with the negative dependence being fairly clear. Despite these caveats, however, there seems general support for the hypothesis the magnitude of past returns influences the actual dynamic mechanism underlying stock prices.

Table 2. Estimates for the slope parameters of the LSTAR model for S&P, FTSE, DJIA and SPI indices, with associated QML standard errors.

	S&P	FTSE	DJIA	SPI
π_{11}	0.1046 (0.027)	0.1766 (0.030)	0.0871 (0.027)	0.1108 (0.035)
π_{12}	-0.0746 (0.027)	-0.0455 (0.034)	-0.0648 (0.026)	
π_{21}	-0.1396 (0.075)	-0.0356 (0.108)	-0.2275 (0.096)	-0.1809 (0.053)
π_{22}	-0.1053 (0.036)	-0.0760 (0.042)	-0.0963 (0.040)	
ϕ	36.474 (6.157)	1777.9 (2310)	308.92 (159.0)	29.97 (42.25)
c	0.0033 (0.000)	0.0049 (0.000)	0.0024 (0.000)	0.0066 (0.000)

In terms of the theoretical model outlined in the previous section, the behaviour summarized by these smooth transition models may be rationalized as follows. For periods of relative calm, when absolute returns are small in magnitude, the behaviour of feedback traders is governed by relatively large thresholds. In these periods fundamentals dominate price behaviour and returns show negative correlation indicating reversion to fundamentals. Once a price breakout occurs, the thresholds of feedback traders narrow as the market moves into a hightened state of excitement. As more feedback trades are triggered price is driven further away from fundamental, resulting in positive correlation between current and past returns. This continues until the demand from fundamental traders forces price to revert and another period of calm ensues.

4. CONCLUSION

This paper has suggested that the absolute magnitude of past returns have a potentially important impact upon the dynamics of the conditional mean of returns. A simple model of trader interaction is proposed, which although minimalist by construction, does appear to produces realistic dynamics. The key element in the model is the fact that past shocks influence trading decision and thus also the dynamic mechanism underlying price formation. To try and relate the model to empirical reality, a logistic smooth transition autoregressive model is estimated for four important equity indices. It is found in general, that these four equity indices do exhibit the kind of pattern suggested by the model of interaction, namely, that when past price movements have been relatively small (large) future dependence is expected to be negative (positive). This is a neglected idea when modeling financial time series where the magnitude of past shocks usually only influence volatility.

5. REFERENCES

Black, F., Noise, *Journal of Finance*, 41, 529-543, 1986.

Cutler, D.M., J.M. Poterba and L.H. Summers, Speculative dynamics and the role of feedback traders, *American Economic Review*, 80, 63-68, 1990.

Chirarella, C., The dynamics of speculative behavior, *Annals of Operations Research*, 37, 101-123, 1992.

Clark, P.K., A subordinated stochastic process model with finite variance for speculative prices, *Econometrica*, 41, 135-56, 1973.

Day, R., and W. Huang, Bulls, bears and market sheep, *Journal of Economic Behaviour and Organization*, 14, 299-329, 1990.

De Grauwe, P., H. Dewachter, and M. Embrechts, *Exchange rate theory: chaotic models of foreign exchange markets*, Blackwell, Oxford, 1993.

De Long, J.B., A. Shleifer, L.H. Summers, and R.J. Waldmann, Noise trader risk in financial markets, *Journal of Political Economy*, 98, 703-738, 1990.

Lamourex, C.G. and W.D. Lastrapes, W.D., Heteroskedasticity in stock return data: volume versus GARCH effects", *Journal of Finance*, 45, 221-228, 1990.

Lux, T., The socio-economic dynamics of speculative markets: interacting agents, chaos, and the fat tails of return distributions, *Journal of Economic Behavior and Organization*, 22, 143-165, 1998.

Teräsvirta, T., Specification, estimation, and evaluation of smooth transition autoregressive models, *Journal of the American Statistical Association*, 89, 208-218, 1994.